EECS205000: Linear Algebra **Spring 2019**
College of Electrical Engineering and Computer Science
National Tsing Hua University

Homework #2
Coverage: Chapter 1–5
Due date: 3 May, 2019

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<u>**Notice**</u>:

1. *Please hand in your answer sheets by yourself to TAs in the class time or to the WCSP Lab., EECS building, R706, before 23:59 of the due date.* ***No late homework will be accepted.***
2. *This homework includes* ***9 problems*** *with* ***100 points plus 5 bonus points.***
3. *Please justify your answers with clear, logical and solid reasoning or proofs.*
4. *You need to* ***print*** *the problem set and answer the problems in the* ***blank boxes*** *after each problem or sub-problm. We provided enough space for every problem. However, if you need more space, you can print it in one-side manner (each page in one side of an A4), and use the back side as an aditional space.*
5. *Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your* ***respect toward the academic integrity.***
6. *Write your name, student ID, email and department on the begining of your ansewr sheets.*
7. *Your* ***legible handwriting*** *is fine. However, you are very welcome to use text formatting packages for writing your answers.*

Name	
Student ID	
Department	
Email Address	

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Problem 1. (10 points) Let $\mathbf{A} = \frac{1}{3}\begin{bmatrix} -2 & 2 & \beta_1 \\ 2 & 1 & \beta_2 \\ 1 & 2 & \beta_3 \end{bmatrix}$. Is it possible to find values of $\beta_1, \beta_2, \beta_3 \in \mathbb{R}$ such that $\mathbf{A}$ to be an orthogonal matrix? Justify your answer by rigorous reasoning.

Problem 2. **(5 points)** Let $\mathbf{b}_1 = [1, 2, 2, 4]^T$, $\mathbf{b}_2 = [-2, 0, -4, 0]^T$, and $\mathbf{b}_3 = [-1, 1, 2, 0]^T$, and let S be the span of these vectors. Apply the Gram-Schmidt process to $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ to obtain an orthonormal basis $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$ for S.

Problem 3. (20 points) Let $\mathbf{A}$ be an $m \times n$ matrix.

(a) (5 points) Show that $N(\mathbf{A}^T\mathbf{A})$=$N(\mathbf{A})$.

(b) (5 points) Show that $rank(\mathbf{A}^T\mathbf{A}) = rank(\mathbf{A})$. (Hint: you may use the rank-nullity theorem, i.e. $dim(N(\mathbf{A})) + rank(\mathbf{A}) = n$ for any $m \times n$ matrix $\mathbf{A}$.)

(c) (10 points) If $\mathbf{A}$ is a 4×3 matrix and $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Let $\tilde{\mathbf{x}}$ be a least-squares solution that minimizes $\|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2$ for $\mathbf{b} = [0, 2, 1, -1]^T$. Find $\mathbf{p} = \mathbf{A}\tilde{\mathbf{x}}$ and give its physical meaning.

Problem 4. (10 points) Assume that the matrix set **M** consists of 2×2 real matrices to form a vector space over $\mathbb{R}^{2\times 2}$.

(a) (5 points) Show that the subspace W consisting of symmetric matrices is a subspace of **M**.

(b) (5 points) Find a basis for W and determine the dimension of W.

Problem 5. (5 points) Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. Find the QR decomposition such that $\mathbf{A} = \mathbf{QR}$ where $\mathbf{Q}$ and $\mathbf{R}$ are the orthogonal and upper triangular matrices, respectively.

Problem 6. (10 points) Let $y = r + sx^2$, where $r, s \in \mathbb{R}$, provide the least squares fit to the points $(x_1, y_1) = (1, 1)$, $(x_2, y_2) = (2, 4)$ and $(x_3, y_3) = (4, 8)$.

(a) (5 points) Find r and s.

(b) (5 points) Find values of y_1, y_2 and y_3 at $x_1 = 1$, $x_2 = 2$ and $x_3 = 4$, respectively, such that the best fitting curve is $y = 0$.

Problem 7. (20 points) Let $\mathbf{A} \in \mathbb{R}^{3\times 5}$. Assume that we performed row operations on $\mathbf{A}$ to convert it to *rref* form, but now we do something different - instead of getting the usual $\mathbf{R} = [\mathbf{I} \quad \mathbf{F}]$, we now reduce it to a matrix in the form of $\tilde{\mathbf{A}} = [\mathbf{F} \quad \mathbf{I}]$. And the row operation of $\mathbf{A}$ was given as follows:

$$\mathbf{A} \xrightarrow{rref} \begin{bmatrix} 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 0 & 1 & 0 \\ 6 & 7 & 0 & 0 & 1 \end{bmatrix} = \tilde{\mathbf{A}}$$

(a) (10 points) Find a basis for $N(\mathbf{A})$.

(b) (10 points) Find a matrix $\mathbf{M}$ so that applying the same row elimination matrix associated with $\tilde{\mathbf{A}}$ to $\mathbf{AM}$ can get the usual *rref* form.

$$\mathbf{AM} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 & 5 \\ 0 & 0 & 1 & 6 & 7 \end{bmatrix}$$

Problem 8. (10 points) Answer the following questions.

(a) (5 points) Let $\mathbf{A} = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$. Find $det(\mathbf{A})$ in terms of x, y, z.

(b) (5 points) Let matrix $\mathbf{A} = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3]$, where $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are vectors in $\mathbb{R}^3$. If $4\mathbf{a}_1 - 3\mathbf{a}_2 + 2\mathbf{a}_3 = \mathbf{0}$, find $det(\mathbf{A})$.

Problem 9. (10 points) Let $\mathbf{C}$ be the cofactor matrix of $\mathbf{A}$, and $\mathbf{C}^T = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -2 & -1 & 2 \end{bmatrix}$. Find the $det(\mathbf{A})$ and $\mathbf{A}$. (Hint: you may use $det(c\mathbf{A}) = c^n det(\mathbf{A})$) for any constant c and $n \times n$ matrix $\mathbf{A}$).

Problem 10. (+5 bonus points) For maximizing your knowledge on Linear Algebra, please provide comments and suggestions of how to improve the teaching quality of this course.